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be regarded simply as a record for comparison with other observers, who may also have already made similar measures.

It may interest some members of the Society who are skilled in astrometry, to be informed that the photometric measures of the relative brightness of all the stars in *Lyra* contained in Dr. Heis' Catalogue, 75 in number, together with two others which ought have been inserted on account of their brilliancy, have now been completed at this Observatory by three sets of five readings on two nights. The consistency of the results is encouraging, and leaves no doubt in my mind of the facility and accuracy of the method devised. I have accordingly arranged for the systematic prosecution of these observations for all the stars visible *nudis oculis* from the Pole to 5° South Declination. This will embrace all the stars recorded in Dr. Heis' Catalogue within the limits prescribed.

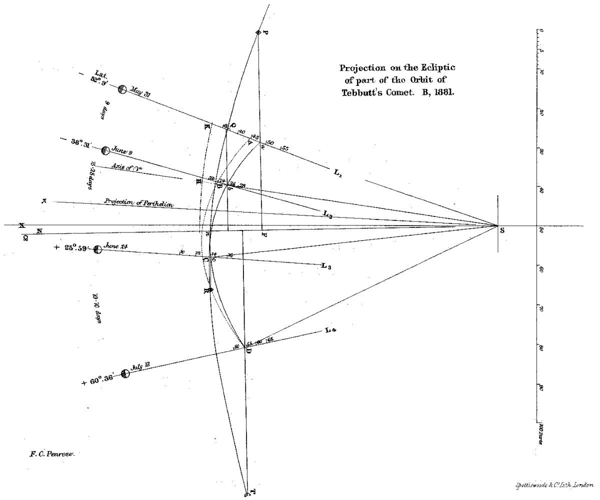
Oxford University Observatory:
1881, *December* 8.

On a Method for Finding the Elements of the Orbit of a Comet by a Graphical Process. By F. C. Penrose, Esq.

It may be interesting to those who are not familiar with the analytical methods of working out the orbit of a Comet to examine a process of graphical construction which will, in many instances at any rate, suffice to give very rapidly a good approximate solution by means which demand only conversance with the methods of practical geometry and sufficient spherical trigonometry to reduce Declination and Right Ascension into latitude and longitude.

I was first led to try if by graphical operations I could get a general idea of the motion of Tebbutt's Comet (1881, *b*), and finding that I succeeded to a greater extent than I had anticipated, I proceeded to attack Schäberle's Comet *c*, with some diffidence, indeed, as it seemed to be giving a good deal of trouble to the computers, and obtained a result which I found compared favourably with the published elements. I venture, therefore, to think that these attempts may be worth laying before the Society.

I should premise that I do not think that much reliance could be placed on a graphical solution of the orbit of a Comet from a few days' observation only, unless it should happen to be near the Earth or to the perihelion; and also that four places are much better than three, although it would be of course theoretically possible to find the elements from three. I will begin with the case of Tebbutt's Comet (Comet *b*, 1881).



Projection on the Ecliptic of part of the Orbit of Tebbutt's Comet. B, 1881.

F. C. Penrose.

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I take four observations reduced to the ecliptic.

	G.M.T.	Longitude.	Latitude.
		° ′ ″	° ′
Cape of Good Hope	May 31·21	69 38 6	−52 9
	June 9·18	74 16 36	−38 31
Greenwich ...	June 24·48	86 19 6	+25 59
	July 13·58	102 37 33	+60 36*

The apparent places are taken, as minute accuracy is not attempted.

The plane of the paper represents the ecliptic, S being the Sun's centre. The positions of the centre of the Earth are calculated for the times given above, and laid down on the diagram at the points entered, with the dates (May 31, June 9, &c.), and from these are laid down the longitude lines (L_1, L_2, &c.). The straight line SX is drawn at 90 degrees from that of the equinoxes. In the scale used six inches represents the Sun's mean distance. For working the solution it is desirable that this scale, divided into 100 parts, should be drawn upon the edge of slip of stout paper or card. The slide rule may also be used with advantage to facilitate the necessary calculations.

The first thing to be done is to obtain a rough approximation to the place of the projection of the Comet where it passes the different longitude lines (L_1 L_2, &c.). To get this it is convenient first to form a short table of numbers proportionate to the time intervals between the dates of the observations, which are respectively about 9, 15·25 and 19·1 days.

9	15·25	19·1
10·6	18·00	22·5
11·2	19·00	23·7
11·8	20·00	25·0
12·3	21·00	26 2
12·9	22·00	27·5

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* The observations are these, namely:—

(1) *Cape of Good Hope.*

	Cape Mean Time.	R.A.	Declination.
	h m s	° ′ ″	° ′ ″
May 31,	6 10 24	75 46 25	−29 42 19
June 9,	5 31 18	77 16 24	−15 44 53

(2) *Greenwich Observations.*

	G.M.T.	h m s	N.P.D. ° ′ ″
June 24,	Transit *sub polo*	5 39 39	40 36 35
July 13,	h m s 13 52 0	...	...

Comet observed in Transit-Circle together with 37 Camelopardi.

δ R.A. (Comet − Star) + $56^{s}·85$

δ N.P.D. (Comet − Star) − $0''·4$

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The mere inspection of the lines L_1 L_2, &c., is sufficient to show that the Comet was passing with a direct course between the Earth and the Sun. Lay the edge of the card or paper scale across these lines and bend it round in such a curve as will cut off upon the scale numbers giving a tolerable approximation to one of the sets in the table given above. For instance, with 20 on the middle arc CB we shall find the other two places fairly represented by A on L_1 and D on L_4; but it will be seen that BA exceeds 11·8, CB being 20. But as the areas about S must be proportional to the times in the projection on the ecliptic as well as in the orbit, it follows that if BA is nearer to S, or lies more obliquely with respect to S than CB, it may exceed the number parallel to 20 on the table; whereas, for the same reason, no such course as the dotted line HK could be chosen.

In this first operation it is almost impossible that the numbers should come out exactly, and it is not worth while to occupy much time upon it. It is, however, important to get fairly proportionate numbers on each side of the middle arc, and in this case the places ABCD, which appear to give the measurements from the scale of 13·5, 20 and 26, will answer for a first start.

The areas of the segments about S must be now examined more carefully. If the points had been correctly chosen

$$\frac{\text{area SBC}}{15{\cdot}3} = \frac{\text{area SCD}}{19{\cdot}1} = \frac{\text{area SAB}}{9};$$

but on comparison we shall find both SCD and SAB to be too large. As respects SCD an obvious improvement will arise from approaching nearer to S, because the lines L_3 and L_4 converge rapidly, and when the points A and C have been advanced to *a* and *c* the areas S *a c* and S *c* D will have a better relation to one another. Also if on L_1 A be shifted to *a* the areas S *a b* and S *b c* will be found to be more nearly adjusted, and the proposed curve now becomes *a b c* D.

Thus far we have only considered the longitudes and the areas resulting from them; but the latitudes should now be examined. And it will be convenient first to mark off upon each of the longitude lines, near the places where the curve crosses them, the points where the height above or below the ecliptic has such values as 40, 45, 50, 55 on L_1, 22, 24, 26, 28 on L_2, &c.—these points being given by the cotangents of the latitude.

We shall see that the amended curve cuts L_1 where this reading is about 47·5, and L_4 at about 56, whilst the horizontal distance between the points is about 52. Half the sum of these heights divided by half the horizontal distance should give a value for the tangent of the angle of inclination,

$$\frac{51{\cdot}75}{26} = \tan 63^\circ\ 20'.$$

From L_2 and L_3, by the same method, we get

$$\frac{19{\cdot}9}{9{\cdot}3} = \tan 64^\circ\ 55'.$$

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I have not tried this Comet on a scale adequate to obtain the best possible graphical approximation, but there is no doubt that the elements above given could be brought nearer the results of calculation which are given below, according to Mr. Hind's parabolic elements, especially as to the date of the perihelion.

T	June 16·457
π	265° 15′ 44″
q	[9·8657]
☊	270° 57′ 46″
i	63 28 46

The other example which I give is Schäberle's Comet, *c*, 1881. Observations on five days have been considered

	G.M.T.	Longitude.	Latitude.
Oxford Radcliffe Observatory	July 31·41	95° 19′ 56″	+22° 34′ 17″
	Aug. 4·42	99 2 24	25 8 9
	10·44	108 5 14	29 49 0
	19·53	137 33 22	36 7 0
Marseilles	Sept. 2·33	199 18 0	17 23 0*

As in the former case, SX is drawn at 90° from the line of the equinoxes, and the same scale is used as in the former diagram.

The first process is to lay down the longitude lines L_1, L_2, L_3, &c., which show that we have now the case of a retrograde orbit, and that the Comet is passing between the Sun and the Earth, and, as the latitudes are all positive, it is, within the above limits, to the north of the ecliptic.

The intervals between the first four observations are respectively 4·01, 6·02, and 9·09 days. From these we determine the following ratios, viz.:—

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* The observations which I have followed are—

Oxford Radcliffe Observatory.

Passage of Comet *sub polo*.

G.M.T.	Observed R.A.	Observed N.P.D. uncorrected for parallax.
h m s	h m s	° ′ ″
July 31, 9 57 43·2	6 28 21·6	44 3 46·1
Aug. 4, 10 32 53·5	6 49 17·7	41 47 13·1
10, 10 29 50·1	7 42 55·6	38 21 32
19, 12 35 2·4	10 23 57·4	40 44 51
Marseilles.		
Sept. 2, 8 0 0	13 37 27	81 31 0

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Observing this difference, it would seem that b and c have been carried rather too near S, because the latitudes at B and C would have given for the value

$$\frac{24+13}{19},$$

giving 62° 50′; but it is undesirable to complicate and confuse the diagram, on so small a scale as is necessary to accompany this paper, with more lines than are shown upon it, and it will suffice for the present purpose to combine the two readings, viz.—

$$\frac{51{\cdot}75+19{\cdot}9}{26+9{\cdot}3} \text{ or } \frac{71{\cdot}65}{35{\cdot}3} = \tan 63^\circ\ 50'.$$

To obtain the line of the nodes we must measure off from b or c a distance corresponding to the cotangent of the angle of inclination, and thus obtain $cn=6{\cdot}82$, and we can now draw SN for the line of the nodes, which makes an angle of 1° with SX, and which, reckoned in longitude, will be 271°.

The next process is to develop the projection so as to get the true figure of the orbit by drawing through the points $a\,b\,c\,$D perpendiculars to SN, and cutting off points upon these lines as determined by the secant of the angle of inclination—viz. such as $\mathrm{PF}=\mathrm{F}a \times \sec 63^\circ\ 50'$.

It will now be convenient to prepare and transfer to tracing paper the arc of a parabola of which the focal distance at the vertex may be taken as Sn.

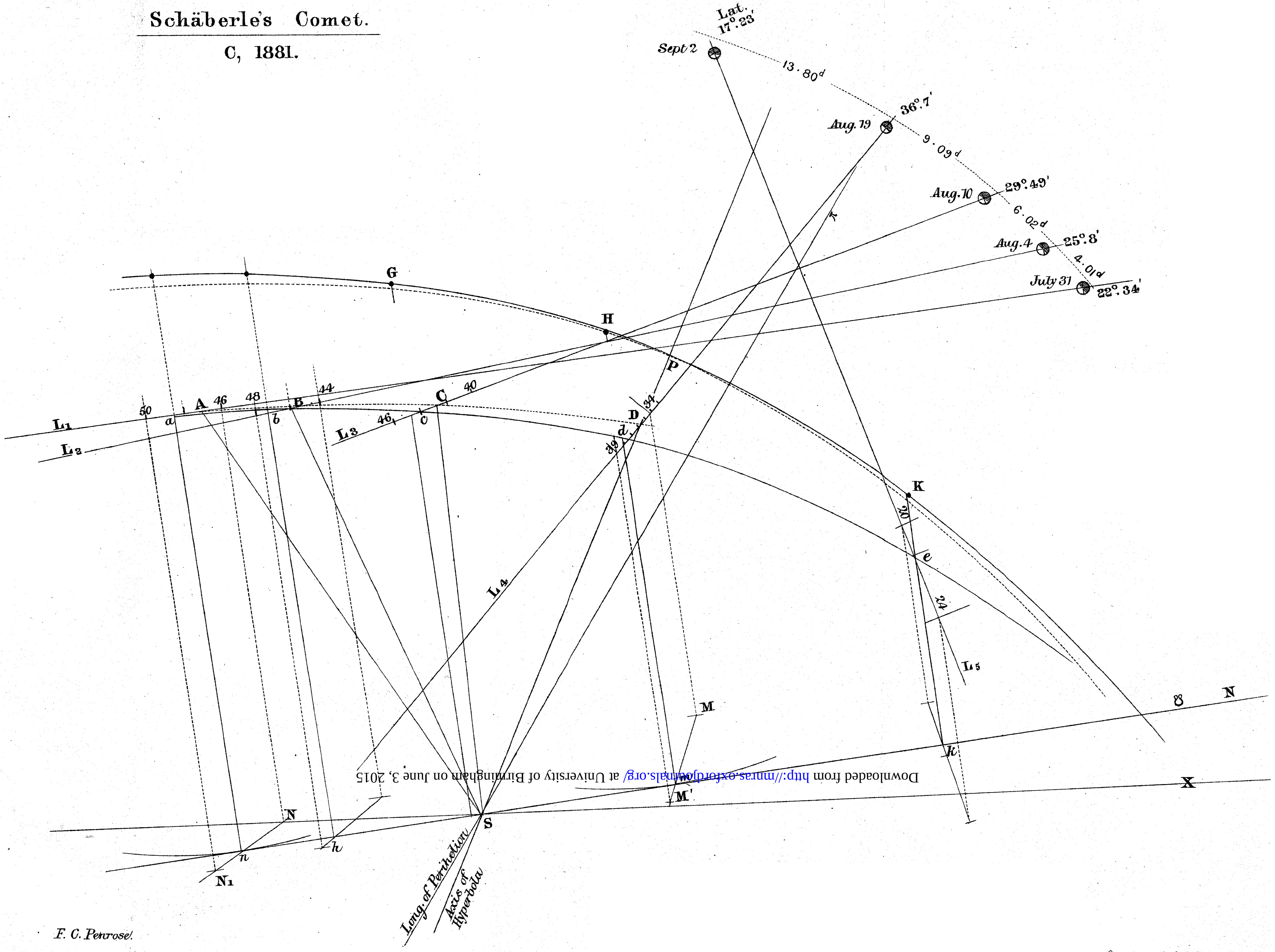
Should it, however, not be obvious that Sn is the proper distance, the way to proceed would be this. Take the points Q and R, and with twice the distance SQ or SR draw arcs of circles, which will intersect in some point beyond S. From this point draw a straight line through S back again to the figure, and it will give the approximate place for the vertex of the parabola, and also determine the parameter. With these parabolic elements construct, as before proposed, on tracing paper an arc of sufficient length to reach from P to T. Then, keeping the focus upon S, turn the tracing about until it makes the best possible coincidence with the points P, Q, R, T, and then it will point out the perihelion, the time of the passage in the developed orbit plane, and, by projection, its longitude on the ecliptic; and it will also give an improved value of the distance.

Following the indications of this small scale figure, the results would be

T	... ...	June 14·70
π	*i.e.* Longitude of Perihelion	266·37
q	Distance	0·73 or [9·8633]
		° ′
☊	... ...	271 0
i	... ...	63 50

Schäberle's Comet.

C, 1881.



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4·01	6·02	9·09
8	12	18·1
10	15	22·6
12	18	27·2
14	21	31·7

If we bend the scale across the longitude lines for a first approximation, we shall find that 12, 18, and 27·2 would agree with the dotted line ABCD pretty nearly; but on drawing a curve through these points and joining them with S, we should find the area ASB to be too small in comparison with BSC, whilst CSD is too large, showing that the projection of the orbit must lie more nearly in the direction *a b c d*. Before that line, however, can be adopted, the latitudes should be examined, and the scales of height above the ecliptic marked upon L_1, L_2, &c.

The line of the nodes can now be drawn approximately. It must pass through S and cannot be far from SN, which direction will follow from the condition that there must be an invariable proportion between the heights above the ecliptic and the length of all such base lines as *a n*, *b h*, &c., this ratio being the tangent of the angle of inclination of the orbit, and this angle will have to be determined simultaneously with the direction of line of the nodes. In getting the angle of inclination primary attention should, in this particular case, be paid to the point C, because a moderate variation in the direction of SN will affect the length of the perpendicular drawn from C in the direction CS very little, and a first trial appears to give for the tangent of the angle of inclination $\frac{43}{51 \cdot 7}$, or 39° 30′; but as this angle will not also satisfy the ratios obtained from the other points, we shall see that C must be moved onwards, as to *c*, where the value of the tangent becomes $\frac{44 \cdot 6}{51 \cdot 7}$, or 41°; and using this value for the inclination, we may proceed to obtain a more exact determination of the line of the nodes.

Looking first at the longitude line L_1, and observing that the approximate curve cuts it near *a*, where the value on the scale formed on L_1 is about 48·5, draw *a n* proportional to the cotangent of 41°. Then from another of the longitude lines (say L_4) draw *d m*, bearing the same ratio to 38, which is the height shown on the scale formed on L_4. These lines *a n* and *d m* may be used as radii for striking arcs of circles, as shown in the diagram, to guide the direction of SN. If the work has been correctly done, *n*, *s* and *m* will lie in the same straight line: if not, draw SN parallel to the line touching the two circular arcs, and take this as the corrected line of the nodes.

On each of the longitude lines, and on each side of the points

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a, *b*, *c*, &c.—viz. from 48 and 50 on L_1, from 44 and 48 on L_2, and so on with regard to the other points—drop perpendiculars on SN proportional to the cotangents of the inclination, and join their extremities as NN_1, MM_1, &c. The line of the nodes will intersect these base lines at *n h m k*; and from the last-named points draw verticals to meet the longitude lines. These will give the corrected places for the projection of the orbit. The areas may now be re-examined, but if the work has been correctly done they will probably be found to satisfy the eye. The method above given has been somewhat more complicated than in the case of Comet *b*, but in that case there were observations available on each side of the ecliptic.

We may now proceed to develop the orbit from the projection, as shown by the dotted lines, by multiplying the verticals already obtained by the secant of the angle of inclination.

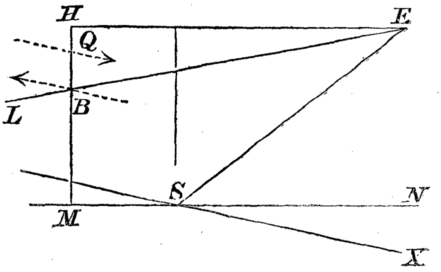
It is quite obvious that the perihelion distance occurs not far from the point H of the developed orbit.

If a circle be drawn through the three points G, H, and K it will show that the nearest point to S will be at or near P. With this distance for its focal length, construct a parabola as in the former case; and after twisting the figure about in the manner already described, we shall obtain the position of the perihelion P and a further correction of the distance. Correct, if necessary, the parabola to the distance so determined, and apply it again to the developed orbit, and it will be found that in this case the points, except those close to the perihelion, will lie in regular sequence outside the parabolic arc, demonstrating that this Comet's orbit must be distinctly hyperbolic. The axis of the curve passes of course through the perihelion, and if projected on to the ecliptic we obtain its longitude.

The elements obtained from this figure are

T	August 22·3
π	147° 30′ 0″
q	0·627 or [9·7973]
☊	95° 55′ 0″
i	41 0 0

In the case of this Comet I have also endeavoured by means of a larger scale, and by the use of trigonometrical formulæ instead of some of the graphic work to obtain a still closer appoximation.



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To explain this method of working, I take one of the points of the former diagram, for instance B. Let the angle

$$\begin{aligned} \text{ESN} &= \alpha \\ \text{SE} &= r \\ \text{EB} &= \rho \\ \text{HB} &= x \end{aligned}$$

Let EH be parallel to SN, and let the angle HEB $= \gamma$, and let l be the latitude of the Comet as seen from E.

It then follows that

$$\rho \tan l = \text{MB} \tan i$$

$$\text{MB} = r \sin \alpha - x, \text{ and } \rho = x \operatorname{cosec} \gamma$$

$$\therefore (r \sin \alpha - x) \tan i = x \operatorname{cosec} \gamma \tan l$$

$$\frac{r \sin \alpha}{x} - 1 = \operatorname{cosec} \gamma \tan l \cot i$$

$$\text{and } x = \frac{r \sin \alpha}{\operatorname{cosec} \gamma \cot i \tan l + 1}.$$

From this it is easy to obtain MB, MH, and SM by calculation, and it will be found more accurate to lay the points which are wanted down by scale than to obtain them graphically from the intersections of lines. If we want the point Q of the developed orbit, it is got from

$$\text{MB} \sec i.$$

When the places in the orbit have been determined in this manner and laid down on the drawing, let the approximating conic section be applied, and it will be seen where the ordinates and abscissæ require modification: this must be done by small variations of the inclination and the line of the nodes; but as these elements affect the different places differently, such modifications of each must be chosen as will best suit the general purpose. To estimate the effect of the different variations we might take the differential coefficients with respect to the angle XSN (which angle may be called ω), and also with regard to i, and apply them to each case. But it is very little more trouble to compute the places directly for slightly different values of i and ω, which has been done to form the following table. The observed places have also been corrected for parallax, &c.

			July 19·164	July 31·41	Aug. 19·53	Sept. 2·33	Sept. 21
i	41° 0′	MQ	·7416	·7419	·5836	·3269	·1130
ω	5 40	SM	·6436	·3146	·2405	·5909	·9005
i	41 0	MQ	·7514	·7468	·5827	·3253	·1123
ω	5 56	SM	·6668	·3223	·2433	·5919	·8964
i	41 15	MQ	·7544	·7482	·5801	·3248	·1123
ω	5 56	SM	·6755	·3261	·2414	·5923	·8966

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From this table the effect of small changes in i and ω upon the different points can be easily ascertained and a choice made of such changes as will best suit an approximating hyperbola. It may, however, be found necessary to reconsider the conic elements, but the same table can again be used for a final adjustment. Working in this manner I have arrived at the following elements of the orbit, which give a very close agreement at all the places that I have tried, and also give sensibly areas proportional to the times.

T	August 22·61
π	$146^\circ\ 44'\ 0''$
q	0·629 [9 79865]
☊	$95^\circ\ 40'\ 0''$
i	40 48 0

And the hyperbola which seems to coincide best with the orbit has for its major axis twelve times the mean distance of the Sun and $e = 1{\cdot}052416$.

The elements which are given in the *Astronomische Nachrichten*, No. 2390,

T	August 22·774
π	$218^\circ\ 36'\ 39''$ or $141^\circ\ 23'\ 21''$ according to the way it is reckoned.
q	[9·80018]
☊	$96^\circ\ 25'\ 48''$
i	139 50 17 or $40^\circ\ 8'\ 43''$ according to the way it is reckoned.

Notes on Sketches of Comet b, 1881. By E. B. Knobel, Esq.

June 24.—The head presented a curious unsymmetrical form: an irregular fan, much developed on the following side, and with little extension on the preceding side; emanating from this in the direction of the tail, was a bright ray, as shown in the sketch made that evening. The brightest portion of the fan was that in front, or nearest the Sun. The nucleus was eccentrically situated in the fan, on the preceding side of the bright ray, which certainly did not radiate from the nucleus. An envelope, apparently of irregular form, surrounded the fan, the curve of which was not symmetrical with the head; exterior to this was the parabolic envelope.

June 29.—The ray seen June 24 had disappeared. The fan was more symmetrical in shape, and the brightest portion was on the preceding side of the nucleus. On this date, at about 11·15, the nucleus passed very close to a 7·4 mag. star, Rad. 1661.